EUROPEAN CENTRAL BANK
EUROSYSTEM

Working Paper Series

Ghassane Benmir, Ivan Jaccard,
Gauthier Vermandel

Green asset pricing

No 2477 / October 2020

Abstract

Climate change is one of the biggest economic challenges of our time. Given the scale of the problem, the question of whether a carbon tax should be introduced is hotly debated in policy circles. This paper studies the optimal design of a carbon tax when environmental factors, such as air carbon dioxide emissions (CO2), directly affect agents' marginal utility of consumption. Our first result is that the optimal tax is determined by the shadow price of CO2 emissions. We then use asset pricing theory to estimate this implicit price in the data and find that the optimal tax is pro-cyclical. It is therefore optimal to use the carbon tax to "cool down" the economy during periods of booms and to stimulate it in recessions. The optimal policy not only generates large welfare gains, it also reduces risk premiums and raises the average risk-free real rate. The effect of the tax on asset prices and welfare critically depends on the emission abatement technology.

Keywords: Climate Change, Compensation Effect, Bond Premium Puzzle, Natural Rate of Interest, Optimal Policy, Welfare.

JEL: Q58, G12, E32.

Non-technical Summary

Climate change is one of the biggest economic challenges of our time. Given the scale of the problem, the question of whether a carbon tax should be introduced is hotly debated in policy circles.

CO2 emissions is a classical example of what economists call "externality". Emissions contribute to climate change, a phenomenon which affects everybody's well-being. The problem is that the adverse effects of emissions are not reflected in market prices. Without a price mechanism, markets fail in the sense that they cannot allocate resources efficiently. This "market failure" in turn leads to excessive CO2 emissions. Government intervention is thus necessary to correct the resulting inefficiency.

This work shows how to design a carbon tax that is optimal from a welfare perspective. We firstly use asset pricing theory to derive the implicit market price of CO2 emissions. We then show that the optimal carbon tax is determined by this implicit price. Next, we use our methodology to compute an estimate of the optimal carbon tax over the business cycle. In our framework, the optimal environmental policy is procyclical. It is therefore optimal to use the carbon tax to "cool down" the economy during periods of booms and to stimulate it in recessions.

Our second main result is that the environmental externality can affect financial markets. In our framework, we find that climate risk reduces the natural rate of interest. This result is relevant for monetary policy because a low natural rate increases the likelihood of hitting the effective lower bound. The reason is that households become more risk averse when firms fail to internalize the damage caused by their emissions. In our simulated economy, low interest rates are thus a consequence of the uncertainty induced by climate change.

A main takeaway is that the effectiveness of the policy critically depends on the ease at which emissions can be abated. The welfare gains from the optimal tax are of a much lower magnitude if the abatement technology is not efficient. Without a well-developed technology, the decline in risk premiums induced by the policy is also much smaller. The success of the policy may therefore critically depend on the timing of implementation. Improving the existing emission abatement technology should probably come first. Once available, an efficient technology would in turn help to mitigate the side effects of the tax, thereby maximizing the welfare gains from the policy.

1 Introduction

The latest available evidence shows that the mean temperature is 1 degree higher than it was in the pre-industrial era. In recent years, this increase in temperature has accelerated and is currently estimated to rise by about 0.2 degrees per decade.[1] The link between carbon dioxide emissions (CO2) and climate change is nowadays clearly established. CO2 emissions are about 20 times higher than they were at the beginning of the 20th century. Moreover, evidence from Antarctic ice cores shows that CO2 emissions have not only risen rapidly, current levels are also the highest in over 400,000 years.[2]

CO2 emissions are not only a low frequency phenomenon, they also exhibit large cyclical fluctuations. A decomposition between trend and cyclical components reveals that CO2 emissions are procyclical and more volatile than GDP (e.g. Doda [2014], Heutel [2012]). Against the background of the ongoing debate concerning a tax on emissions, these large cyclical fluctuations raise several important questions. In particular, are these strong cyclical fluctuations desirable from a welfare perspective? And how should the optimal carbon tax vary over the business cycle?

This paper addresses these questions by studying the optimal carbon tax in the presence of an environmental externality. The novelty of our approach is to investigate the link between asset pricing theory —in particular the stochastic discount factor (SDF)— and climate policies. The SDF is a key building block of modern asset pricing theory (e.g. Cochrane [2011]). Our main contention is that it also has a critical impact on the level and cyclicality of the optimal carbon tax.

Following Stokey [1998], Acemoglu, Aghion, Bursztyn, and Hemous [2012] and Golosov, Hassler, Krusell, and Tsyvinski [2014], among others, environmental considerations are captured by introducing an externality into the utility function. Apart from a few exceptions (see for instance Michel and Rotillon [1995]), most papers in this literature use a separable specification that implies no direct link between the environment and marginal utility of consumption. Our innovation is to study a model in which the presence of an environmental externality raises households' willingness to consume goods.

Our approach can be motivated by the effect of climate change on consumption. As documented by Abel, Holloway, Harkey, Meier, Ahl, Limaye, and Patz [2018] and Mansur,

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[1] Pachauri, Allen, Barros, Broome, Cramer, Christ, Church, Clarke, Dahe, Dasgupta, et al. [2014].

[2] The Economist (2019). "Briefing Climate Change", Sept. 21st-27th.

Mendelsohn, and Morrison [2008], one perverse effect of climate change is to increase the use of electricity. Higher levels of emissions cause climate change, which in turn increases the need to consume electricity to cool homes. This complementarity between climate change and consumption can be illustrated by the exponential increase in the use of air-conditioning in the last decades.[3] Projections by the International Energy Agency also suggest that this is only the beginning, as the demand for air-conditioning is expected to triple by 2050.[4]

This therefore suggests the existence of a compensation effect of climate change (e.g. Michel and Rotillon [1995]). As greenhouse gas emissions increase, the need to consume electricity as well as other goods to mitigate the effect of climate change becomes more pressing. Since the electricity sector is in turn one of the largest sources of emissions, this doom loop of climate change can have macroeconomic implications.

From a finance perspective, whether marginal utility is affected by environmental factors also has important implications. Indeed, the SDF —the ratio of future relative to current marginal utility—is at the core of modern asset pricing theory. Consequently, if environmental factors modify agents' marginal utility of consumption, they will also affect the pricing of risky and safe assets. This compensation effect of climate change therefore implies a potential role for green factors in asset pricing models.

The effect of the environmental externality on marginal utility is then obtained by adopting an approach similar to that employed in the seminal contribution of Campbell and Cochrane [1999]. In our case, however, it is the current stock of CO2 emissions rather than past levels of consumption that negatively affects utility. Moreover, following Heutel [2012], the stock of emissions is modelled as a slow-moving variable whose level depends on the quantity of emissions. As in Campbell and Cochrane [1999], this specification implies that risk aversion increases when the distance between consumption and the externality declines. Another motivation for adopting this particular specification is that it will allow us to generate realistic fluctuations in the SDF.

Relative to the endowment economy approach (e.g. Lucas Jr [1978]), another difference is that we study the environmental externality in a production economy following the seminal contribution of Jermann [1998]. We then derive the optimal tax by comparing the decentral-

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[3] The Economist (2018). "Air-conditioners do great good, but at a high environmental cost". August 25th.

[4] International Energy Agency (2018). "Air conditioning use emerges as one of the key drivers of global electricity-demand growth". News, May 15th 2018.

ized equilibrium with the planner's problem, as usually done in the environmental literature (e.g. Xepapadeas [2005]) or in Ljungqvist and Uhlig [2000] for the case of a consumption externality.

Following Heutel [2012] and Nordhaus [2008] among others, we introduce an abatement technology that firms can use to reduce their carbon footprint. Even when available, firms do not use this technology if emissions are not taxed. The abatement technology diverts resources from production. Consequently, profit-maximizing firms have no incentives to reduce emissions unless they are forced to do so.

Our first main result is that the optimal tax is determined by the shadow value of CO2 emissions. We show that this implicit price can be expressed as the infinite discounted sum of the marginal disutility caused by emissions. This discounted sum is in turn critically affected by the SDF that agents use to price assets. This result therefore highlights the importance of asset pricing considerations for the design of an optimal environmental tax.

Imposing a tax on emissions restores the first-best allocation by encouraging firms to use the abatement technology. Abating carbon emissions is costly for firms. From the point of view of the social planner, it is therefore optimal to set the cost of abating emissions faced by firms to its social cost. Since it represents the social cost of emissions, the optimal tax is thus determined by the implicit price of CO2 emissions.

Our second main result is that slow movements in the stock of CO2 can have significant financial market implications. Of particular relevance to central banks is the finding that environmental externalities affect the natural rate of interest. Climate change reduces the natural rate of interest, thereby increasing the likelihood of hitting the effective lower bound.

The intuition for this result is that the environmental externality generates time-variation in risk aversion, as in a model with external habits. In other words, when firms fail to internalize the damage caused by their emissions, households become more risk averse. This increase in risk aversion raises the risk premium demanded by investors and induces precautionary saving. This stronger precautionary motive in turn explains the effect on the natural rate of interest.

Next, we show that introducing an optimal environmental tax reduces risk premia and increases the natural rate of interest. Under our baseline scenario, the tax reduces the premium on a long-term bond by around half and increases the natural rate by around 2 percent.

This result can be explained by the effect of the optimal policy on risk aversion. A tax on production reduces output, and hence consumption as well as emissions. The key is that the decline in emissions causes a reduction in the externality that exceeds the fall in consumption. The resulting increase in this distance between consumption and the externality, or "surplus consumption" in the case of habits (e.g. Campbell and Cochrane [1999]), in turn reduces risk aversion.

Although consumption declines, the optimal tax generates large welfare gains. Under our benchmark calibration, this result can be explained by the large decline in emissions induced by the policy. The magnitude of this gain in turn depends on how firms react to the carbon tax. A profit-maximizing firm increases abatement until the marginal cost of abating emissions equals the marginal benefit. Under the optimal policy, the tax incentivizes firms to use the abatement technology to reduce the burden of the tax. This incentive to reduce emissions therefore explains the large welfare gain that we obtain.

The effect on welfare critically depends on the efficiency of the abatement technology available in the economy. If the technology is not sufficiently well-developed, the distortion caused by the tax can be sizeable. Indeed, if firms cannot circumvent the tax by abating emissions, their only choice is to reduce production. In this case, the tax generates a smaller decline in emissions, which in turn reduces the welfare gains from the policy.

The effect of the optimal policy on asset prices also crucially depends on the abatement technology. In this model, this can be explained by the impact of the tax on risk aversion. Indeed, a less developed technology reduces the decline in the stock of emission induced by the carbon tax. Consequently, a smaller increase in the distance between consumption and the externality can be achieved if the technology is inefficient. This in turn implies a lower reduction in risk aversion, which causes higher risk premia and lower real interest rates.

Our third main result is that the optimal tax is procyclical. As in Ljungqvist and Uhlig [2000], it is therefore optimal to "cool down" the economy during periods of booms and to stimulate it in recessions. Estimating the model using higher-order perturbation methods allows us to estimate the implicit price of carbon. Our approach can therefore be used to provide an estimate of the optimal carbon tax over the business cycle. As illustrated in Figure 2, it would have been optimal to progressively increase the tax in the run-up to the financial crisis and to reduce it sharply when the financial shock hit.

The intuition for this result is that the externality induces fluctuations in risk aversion that are excessive. As in a model with external habits and time-varying risk aversion (e.g. Campbell and Cochrane [1999]), the externality is beyond agents' control. By internalizing the effect of emissions on utility, the policy allows the planner to find an optimal trajectory for both consumption and the stock of emissions. Controlling both variables at the same time in turn reduces the variations in "surplus consumption" that are unnecessary from a welfare perspective. These lower fluctuations in turn imply more moderate variations in risk aversion.

During recessions, this is achieved by lowering the carbon tax. A decline stimulates emissions. This effect contributes to increase risk aversion by reducing the distance between consumption and the externality. The key is that, as in the data, the stock of emissions moves very slowly over time. Since the impact of the policy on consumption is more immediate, a tax cut generates an increase in consumption that exceeds the increase in the stock of emissions. The optimal policy therefore allows the planner to mitigate the surge in risk aversion that occurs in recessions.

As pointed out by Bansal, Kiku, and Ochoa [2019] and van den Bremer and van der Ploeg [2019], there is evidence that climate change risk could already be reflected in current equity prices. In Bansal et al. [2019], this link is explored in a model in which climate change is a source of long-run risk (e.g. Bansal and Yaron [2004]). The long-run risk approach relies on Epstein-Zin-Weil preferences (e.g. Epstein and Zin [1989]; Weil [1989]; Weil [1990]). In van der Ploeg, Hambel, and Kraft [2020], the optimal carbon tax is derived in an endogenous growth model. One main result is that climate disaster risk has significant asset pricing implications.

In our case, environmental factors affect financial markets through the effect of the externality on attitudes towards risk. Everything else equal, the key is that an increase in the stock of emissions increases risk aversion. Whereas it is difficult to test this hypothesis in the data, recent results in the psychology literature provide some indirect support.

First, in this literature, it is well-established that air pollution tends to increase anxiety. A recent review of the evidence on the link between air pollution and anxiety is provided in Lu [2020]. Air pollution is in turn strongly correlated with CO2 emissions. Second, there is evidence that anxiety and risk aversion are tightly linked. For instance, according to Charpentier, Aylward, Roiser, and Robinson [2017], more anxious individuals exhibit a reduced propensity to take risks. The authors of this study argue that this result is driven by risk aversion, and not loss aversion.

Such an effect of air pollution on risk aversion is also consistent with the findings documented by Levy and Yagil [2011]. Indeed, they find a negative correlation between air pollution and stock returns. Their interpretation is that air pollution has negative mood effects. Since experimental studies in psychology in turn relate bad mood to increased risk aversion, they argue that air pollution could affect stock returns.

More generally, there is increasing evidence that air pollution could affect economic choices. For instance, Chang, Huang, and Wang [2018] find that air pollution has a significant effect on the decision to purchase or cancel health insurance.

2 The model

Consider a business cycle model characterized by discrete-time and an infinite horizon economy populated by *firms* and *households*, which are infinitely lived and of measure one. In this setup, production by firms induces an environmental externality through emissions. The latter affects the welfare of households by decreasing the utility stemming from consuming goods. Firms do not internalize the social cost from their emissions of CO2. This gives rise to a market failure that opens the door for optimal policy intervention.

As the contribution of the paper lies in the role of the environmental externality in shaping risk behavior of investors, we start by presenting the accumulation of emissions in the atmosphere. We then explain how the environmental externality affects households' behavior.

2.1 Firms and emissions

Following standard integrated assessment models (IAM) (see Nordhaus [1991] or Nordhaus and Yang [1996]), a major part of the accumulation of carbon dioxide and other greenhouse gases (GHGs) in the atmosphere results from the human activity of economic production. Therefore, we employ a similar law of motion as IAM to describe the concentration process of carbon dioxide in the atmosphere:

$$x_{t+1} = \eta x_t + e_t, \tag{1}$$

where x_{t+1} is the concentration of gases in the atmosphere, $e_t \geq 0$ is the inflow (in kiloton) of greenhouse gases at time t, and $0 < \eta < 1$ the linear rate of continuation of CO_2-equivalent emissions that enter the atmosphere on a quarterly basis.[5] Anthropogenic emissions of CO_2 result from both economic production and exogenous technical change:

$$e_t = (1 - \mu_t)\, \varphi_1 y_t^{1-\varphi_2} \varepsilon_t^X \Psi_t. \tag{2}$$

Here, variable $1 \geq \mu_t \geq 0$ is the fraction of emissions abated by firms, y_t is the aggregate production of goods from firms, variable ε_t^X is an AR(1) exogenous shock on the carbon intensity of firms and Ψ_t is a technical change trend in carbon intensity.[6]

This functional form of emissions allows to take into account both low and high frequency variations in CO_2 emissions. For the high frequency features of the emissions data, the term $\varphi_1 y_t^{1-\varphi_2}$ denotes the total inflow of pollution resulting from production, prior to abatement. In this expression, parameters φ_1, $\varphi_2 \geq 0$ are two carbon intensity parameters that respectively pin down the steady state ratio of emissions-to-output as well as the elasticity of emissions with respect to output over the last century. While φ_2 is set to 0 in Nordhaus [1991], we follow Heutel [2012] and allow this parameter to be positive to capture potential nonlinearities between output and emissions. Note that for $\varphi_2 < 1$, the emissions function exhibits decreasing returns.

For low frequency dynamics of CO2 emissions, these are jointly determined by the trend on output and the carbon efficiency trend Ψ_t. The latter is necessary to capture the long term process of decoupling between output growth and emission growth. As documented by Newell, Jaffe, and Stavins [1999], this trend can be interpreted as an energy-saving technological change that captures the adoption of less energy intensive technologies on capital goods. As Nordhaus [1991], we assume that this trend Ψ_t is deterministic and grows at a constant rate: $\Psi_t = \gamma_E \Psi_{t-1}$, where γ_E is the growth rate.

The remaining set of equations for firms is rather standard and similar to Jermann [1998]. In particular, the representative firm seeks profit maximization by making a trade-off between

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[5] One limitation is that we do not consider emissions by the rest of the world (ROW). At the same time, US and ROW emissions are strongly correlated at a business cycle frequency. Moreover, the US accounts for 1/3 of total anthropogenic emissions.

[6] For simplicity, we assume that the exogenous trend Ψ is not affected by abatement μ.

the desired level of capital and labor:

$$y_t = \varepsilon_t^A k_t^\alpha \left(\Gamma_t N_t\right)^{1-\alpha}, \tag{3}$$

where k_t is the capital stock with an intensity parameter $\alpha \in [0,1]$, N_t is labor, and ε_t^A is total factor productivity shock that evolves as follows: $\log\left(\varepsilon_t^A\right) = \rho_A \log\left(\varepsilon_{t-1}^A\right) + \eta_t^A$ with $\eta_t^A \sim N(0, \sigma_A^2)$. Long term economic growth also results from a labor productivity augmenting trend $\Gamma_t = \gamma_A \Gamma_{t-1}$ that affects labor in the production function.

Firms maximize profits:

$$d_t = p_t y_t - w_t N_t - r_t k_t - f\left(\mu_t\right) y_t - e_t \tau_t, \tag{4}$$

where p_t is the real marginal cost of production,[7] w_t denotes the real wage, r_t is the cost of renting capital, $f\left(\mu_t\right)$ is the abatement cost function, and $\tau_t \geq 0$ is a possible tax on GHGs emissions implemented by the fiscal authority. The abatement cost function is taken from Nordhaus [2014], where $f\left(\mu_t\right) = \theta_1 \mu_t^{\theta_2}$. In this expression, $\theta_1 \geq 0$ pins down the steady state of the abatement while $\theta_2 > 0$ is the elasticity of the abatement cost to the fraction of abated GHGs. This function $f\left(\mu_t\right)$ relates the fraction of emissions abated to the fraction of output spent on abatement, where the price of abatement is normalized to one.

2.2 *Households and the environmental externality*

We model the representative household by using a CRRA utility function where the household chooses consumption expenditures, investment as well as its holding of long-term government bonds. Following Stokey [1998], Acemoglu et al. [2012] and Golosov et al. [2014], we introduce the environmental externality into the utility function. However, instead of considering an additive specification, we assume that the marginal utility of consumption is affected by the externality.

Given our focus on asset prices, we choose a specification similar to that employed in the seminal contributions of Campbell and Cochrane [1999]. As will become clear, adopting this particular specification will dramatically improve the model's ability to generate realistic asset pricing implications.

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[7] Note that the real marginal cost is normalized to one in a standard real business cycle model, but becomes non-unitary and time-varying under positive abatement costs.

The utility of the representative agent depends on the distance between consumption and the externality:

$$\max_{\{c_t,k_{t+1},i_t,b_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{(c_t - \phi_t x_t)^{1-\sigma}}{1-\sigma}, \tag{5}$$

where E_0 is the expectation operator conditioned upon information at time 0, $\beta \in [0,1]$ the time discount factor, and $\sigma > 0$ the curvature parameter. The parameter ϕ_t represents the sensitivity of utility to a rise in CO_2 concentration in the atmosphere, which is denoted by x_t.[8] It could also be interpreted as the the proportion of consumers affected by the damage caused by CO2 emissions. Furthermore, the externality is a predetermined variable that moves slowly over time. This is to account for the possible long-term effects of decisions made in the past, which have consequences (possibly irreversible) in the future. This assumption has important implications for optimal choices, which we discuss in the following paragraphs.

First, from a consumer perspective, consumption and the stock of CO2 emissions can be interpreted as complements. As a result, the marginal utility of consumption increases in CO2 concentration, so households are more willing to consume when GHG concentration is high. This mechanism, pioneered by Michel and Rotillon [1995], is referred to as the *compensation effect*: households consume more to compensate the drop in utility due to an increase in emissions.

Second, this environmental externality in the utility function also has important asset pricing implications. To illustrate this point, let us define as Campbell and Cochrane [1999] the consumption surplus ratio, $s_t = (c_t - \phi_t x_t)/c_t$. When the surplus falls in cyclical downturns, investors require a higher expected return compared to a standard CRRA utility function where $\phi = 0$. Under these preferences, the coefficient of relative risk aversion is given by $-(u''_c/u'_c)c_t = \sigma/s_t$. Therefore, an increase in the stock of emissions reduces the surplus, which in turn increases risk aversion.

The budget constraint of the representative household is given as follows:

$$w_t N_t + r_t k_t + b_t = c_t + i_t + p_t^B (b_{t+1} - b_t) + T_t, \tag{6}$$

where the left hand-side of this equation denotes the household's different sources of income.

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[8] Note that c_t and x_t are not growing at the same rate in the deterministic steady state of the model. To induce a balanced growth in each side of the utility function, we assume that ϕ_t is putting variable pollution on the same slope of growth as consumption: $\phi_t = \phi(\Psi_t \Gamma_t^{1-\varphi_2})^{-1}$. Therefore, the detrended utility function is given by $\frac{\Gamma_t^{1-\sigma}}{1-\sigma} (\tilde{c}_t - \phi \tilde{x}_t)^{1-\sigma}$.

Total income is firstly comprised of a labor income (with inelastic labor supply N_t). The capital stock that is rented to firms is denoted by k_t, where r_t is the rental rate of capital. Every period, the agent receives an income from holding a long-term government bond, b_t. The representative household firstly spends his or her income on consumption and investment goods, which are denoted by c_t and i_t, respectively. The price at which newly issued government bonds are purchased is denoted by p_t^B. Finally, we assume that the government levies a lump-sum tax, which we denote by T_t.

The accumulation of physical capital is given by the following law of motion:

$$k_{t+1} = (1 - \delta)k_t + \quad \left(\varepsilon_t^I \frac{i_t}{k_t}\right) k_t, \tag{7}$$

where $\delta \in [0, 1]$ is the depreciation rate of physical capital, $\psi(\bullet)$ is an adjustment cost function on investment, and ε_t^I is an exogenous shock process as in Christiano, Motto, and Rostagno [2014]. This shock can be interpreted as an investment shock that captures financial frictions associated with asymmetric information or costly monitoring. As in Jermann [1998], each unit of investment yields up to $\left(\varepsilon_t^I i_t / k_t\right) k_t$ units of physical capital with $\psi(x) = \frac{1}{1-\epsilon} b_1 x^{1-\epsilon} + b_2$, where b_1 and b_2 are two scale parameters and $\epsilon > 0$ is the intensity of the cost function. Note that is also the elasticity of Tobin's Q to a change in the investment-to-capital ratio of firms. Thus ε_t^I also captures some fundamental changes in Tobin's Q which are not driven by the investment-capital structure of firms.

2.3 Government and market clearing

The government finances its expenditures by issuing a bond and by collecting taxes. The government budget constraint is given as follows:

$$g_t + b_t = p_t^B (b_{t+1} - b_t) + T_t + \tau_t e_t, \tag{8}$$

where public expenditures are denoted by g_t, and where T_t is a lump-sum tax. The revenue is composed of government bonds b_{t+1}, issued on financial markets to households, while the term $\tau_t e_t$ denotes the revenues obtained from the implementation of an environmental tax on emissions set by the social planner. In this expression, e_t and τ_t denote the level of emissions and the tax, respectively. As in any typical business cycle model, government spending is exogenously determined and follows an AR(1) process: $g_t = \bar{g}\varepsilon_t^G$ with $\log \varepsilon_t^G =$

$\rho_G \log \varepsilon^G_{t-1} + \eta^G_t$, $\eta^G_t \sim N(0, \sigma^2_G)$ and $\bar{g}$ denote the steady state amount of resources that is consumed by the government. This shock accounts for changes in aggregate demand driven by both changes in public spending and the trade balance.

The resource constraint of the economy reads as follows:

$$y_t = c_t + i_t + g_t + f(\mu_t)\, y_t. \tag{9}$$

Finally, for asset pricing variables, we compute the risk-free rate and the conditional risk premium respectively as:

$$1 + r^F_t = \{E_t m_{t,t+1}\}^{-1}, \tag{10}$$

$$E_t(r^B_{t+1} - r^F_t) = E_t((1 + p^B_{t+1})/p^B_t - (1 + r^F_t)), \tag{11}$$

where $m_{t,t+1} = \beta \{\lambda_{t+1}/\lambda_t\}$ is the stochastic discount factor.

3 Welfare theorems under environmental preferences

In this section, we derive the optimal tax by comparing the decentralized equilibrium with the planner's problem.

3.1 The centralized economy

We start by characterizing the first-best allocation by considering the optimal plan that the benevolent social planner chooses so as to maximize welfare. This equilibrium provides the benchmark against which the allocation obtained in the decentralized economy should be compared.

Definition 1 *The optimal policy problem for the social planner is to maximize total welfare in Equation 5 by choosing a sequence of allocation for quantities* $\{c_t, i_t, y_t, g_t, \mu_t, e_t, k_{t+1}, x_{t+1}\}$, *for given initial conditions for the two endogenous state variables* k_0 *and* x_0, *that satisfies equations (1), (2), (3), (7), and (9).*

Define λ_t as the time t marginal utility of consumption, q_t as the shadow value of capital and ϱ_t as the Lagrangian multiplier on the production function (note that both q_t and ϱ_t are expressed in terms of marginal utility of consumption). The first-order conditions for this problem are given as follows:

$$\lambda_t = (c_t - \phi_t x_t)^{-\sigma}, \tag{12}$$

$$1 = b_1 \varepsilon^I_t q_t \left(\varepsilon^I_t \frac{i_{t+1}}{k_{t+1}} \right)^{-\epsilon}, \tag{13}$$

$$q_t = E_t \left\{ m_{t,t+1} \bigg[\left((1-\delta_K) + \quad \left(\varepsilon^I_{t+1} \frac{i_{t+1}}{k_{t+1}} \right) - b_1 \left(\varepsilon^I_{t+1} \frac{i_{t+1}}{k_{t+1}} \right)^{1-\epsilon} \right) q_{t+1} + \varrho_{t+1} \alpha \frac{y_{t+1}}{k_{t+1}} \bigg] \bigg] \right\}, \tag{14}$$

where the stochastic discount factor is the ratio of future relative to current marginal utilities of consumption: $m_{t,t+1} = \beta \left\{ \lambda_{t+1} / \lambda_t \right\}$.

Letting V^E_t denote the Lagrangian multiplier (expressed in units of marginal utility of consumption) on equation (2), the first-order conditions with respect to the firm's optimal choice of output and abatement are given as follows:

$$\varrho_t = 1 - f(\mu_t) - V^E_t (1-\varphi_2)\, e_t / y_t, \tag{15}$$

$$V^E_t e_t / (1-\mu_t) = f'(\mu_t)\, y_t. \tag{16}$$

The Lagrange multiplier ϱ_t is usually interpreted as the marginal cost of producing a new good, while V^E_t is the social planner's value of abatement. Thus, equation (15) highlights the key role of emissions in shaping price dynamics: the production of one additional unit of goods reduces the profits of firms by $f(\mu_t)$ but is partially compensated by the marginal gain from emitting GHGs in the atmosphere. Notice that if abatement effort is zero, the marginal cost of production is one, as in the standard RBC model. The second equation (16) is a standard cost-minimizing condition on abatement: abating CO2 emissions is optimal when its marginal gain (left hand-side of equation 16) equals its marginal cost (right hand-side of the same equation).

Two remaining first-order conditions on each environmental variables, namely x_t and e_t, are necessary to characterize the decision rules of the social planner:

$$V_t^X = E_t\left\{m_{t,t+1}\left(\phi_{t+1} + \eta V_{t+1}^X\right)\right\}, \tag{17}$$

$$V_t^E = V_t^X. \tag{18}$$

Recall that V_t^E is the Lagrange multiplier on emissions in equation (2) while V_t^X is the Lagrange multiplier on the law of motion of GHGs in equation (1). Variable V_t^X can be interpreted as the social cost of carbon (SCC) for the society, as it measures the economic loss caused by a marginal increase in carbon emissions. As recently suggested by Shayegh, Bosetti, Dietz, Emmerling, Hambel, Jensen, Kraft, Tavoni, Traeger, and Van der Ploeg [2018], this approach for calculating SCC, which relies on computing marginal abatement cost (as opposed to using constrained cost-benefit analysis), allows to better capture and assess the impact of uncertainty and risk management in climate change scenarios.

Equation (17) shows that this social cost can be interpreted by an asset pricing formula. The first term—denoted $E_t m_{t,t+1}\phi_{t+1}$—is the discounted utility loss incurred by society of a marginal increase in the stock of emissions in the atmosphere. The second term—$\eta E_t\{m_{t,t+1}V_{t+1}^X\}$—is the continuation value of the discounted utility loss caused by emissions, which remains in the atmosphere with a probability η. The second equation is the internal cost of GHG emissions for firms, where V_t^E is the marginal cost for a firm for emitting one kiloton of carbon. This cost is firstly determined by the social cost of carbon, which is given by equation (17). The second component of this marginal cost is determined by the tax on carbon emissions implemented by the social planner. Note that V_t^E is interpreted as the value of effort from reducing CO2 emissions.

Definition 2 *The inefficiency wedge induced by the environmental externality is defined as the gap between the social cost of carbon and the marginal cost of emissions:*

$$\varpi_t = V_t^X - V_t^E. \tag{19}$$

When the social cost of carbon is perfectly internalized by the society, the optimal abatement effort in (18) is set such as to equalize the marginal cost of emissions to the social cost of carbon for the society. In this case, it is optimal for firms and the society to spend a fraction of resources to reduce CO2 emissions by using the abatement technology $f(\mu_t)$.

Proposition 1 *In a centralized equilibrium, the social cost of carbon is perfectly internalized by the planner. Therefore, the marginal cost of emissions is equal to the social cost of carbon. This implies (from the previous definition) a first-best allocation with an inefficiency wedge* $\varpi_t = 0$.

The resulting equilibrium is optimal because the social cost of the externality is perfectly internalized by the society. As a consequence, the inefficiency wedge from carbon emissions is zero. In the following section, we show that this optimum is not reached in a decentralized equilibrium with profit-maximizing firms.

3.2 The competitive equilibrium

We now describe the competitive equilibrium resulting from economic decisions taken by households and firms separately, with no centralization scheme. This decentralized economy is also referred to as the competitive or *laissez-faire* equilibrium where social preferences for carbon are different across firms (J_t^X) and households (V_t^X). We propose the following definition to characterize this economy.

Definition 3 *The laissez-faire equilibrium is defined as a competitive equilibrium in which the environmental tax on carbon emissions* τ_t *is set to* 0*. Households maximize utility in Equation 5 under constraints (6) and (7). Firms maximize profits (4) under constraints (2) and (3).*

Relative to the efficient equilibrium, the difference in this situation is that firms maximize profits and no longer consider the stock of CO2 emissions as a control variable. This implies that firms and households exhibit different preferences regarding carbon emissions. As a result, the social cost of carbon for firms differs from that obtained in the centralized economy (i.e. $J_t^X < V_t^X$). Since emissions are costly to abate, and given that firms do not internalize the effect of their emissions on consumers, the cost of carbon emissions for firms is zero:

$$J_t^X = 0. \tag{20}$$

In contrast, the social cost of carbon for households, which we denote V_t^X, is given as follows:

$$V_t^X = E_t \left\{ m_{t,t+1} \left(\phi_{t+1} + \eta V_{t+1}^X \right) \right\} . \tag{21}$$

In this context, a market failure emerges since the social value of carbon differs across emitters of carbon and agents experiencing the social loss.

In the absence of an environmental policy, this decentralized equilibrium is referred to as the *laissez-faire* equilibrium. Since emissions are not taxed, this implies that the shadow cost for a firm to emit CO2 in the atmosphere is zero:[9]

$$J_t^E = 0. \tag{22}$$

Under this setup, firms are simply cost-minimizing by optimally choosing zero abatement spending. Since the cost of releasing CO2 is nil, firms have no incentive to allocate resources to use the abatement technology $f\left(\mu_t\right)$ to reduce emissions. The socially optimal quantity of abatement is not implemented, as the equilibrium abatement share is zero in the laissez-faire equilibrium:

$$\mu_t = 0. \tag{23}$$

Consequently, the marginal cost of production ϱ_t is similar to that obtained in any typical real business cycle model. In terms of the notation introduced in 3, this in turn implies an environmental inefficiency wedge that differs from zero:

$$\varpi_t = V_t^X - J_t^E = V_t^X. \tag{24}$$

Therefore, CO2 emissions create a market failure through an environmental externality. As a result, the first welfare theorem breaks down because the competitive equilibrium does not coincide with the social planner's outcome. The externality, measured by the inefficiency wedge ϖ_t, distorts the equilibrium and gives rise to a deadweight loss proportional to V_t^X. Note that the first welfare theorem applies only if the environmental policy has no effect on preferences, which is the case only if $\phi_t = 0$ (for $t > 0$).

3.3 Environmental policy

In the presence of an environmental externality measured by $\varpi_t > 0$, the social value of carbon differs across agents. This market failure opens the door for government intervention. Thus, the government can use a policy tool to eliminate this externality and make the

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[9] The optimality conditions corresponding to the *laissez-faire* equilibrium are shown in section 10.2.

allocation obtained in the *laissez-faire* economy coincide with that of the social planner. In particular, the government can introduce a tax, denoted τ_t, on GHGs emissions paid by firms. This policy tool has two interpretations. First, this tool can be interpreted as a tax on carbon emissions, in the same spirit as a standard Pigouvian tax that aims to force firms to internalize the social cost of carbon emissions on households' utility, thereby correcting the market failure (i.e. the negative externality) by setting the tax to the social cost of carbon emissions. An alternative interpretation of this policy instrument is the creation of a carbon emissions market (i.e. a carbon permits market), whose market price is decided by the government to regulate the quantity of emissions. The optimal value for this instrument can be directly computed from a Ramsey optimal problem. Comparing the social planner's solution to the competitive equilibrium, we find the following proposition:

Proposition 2 *The first-best constrained allocation can be attained by using the instrument* τ_t *in order to close the inefficiency gap (i.e.* $\varpi_t = 0$*). This condition is achieved by setting the carbon tax such that:*

$$\tau_t = V_t^X.$$

As shown in section 10.3, in appendix C, setting the tax rate to V_t^X ensures that the first-order conditions under the competitive and the centralized equilibria coincide. This result is rather intuitive. In the absence of an environmental policy, abatement deteriorates profits, so firms are not willing to bear this cost unless an enforcement mechanism is implemented. Therefore, if the government imposes a price on carbon emissions by choosing the optimal tax (either quantity or price based as discussed in Weitzman [1974]), the policy exactly triggers the desired level of abatement. This environmental policy incentivizes firms to internalize the effect of emissions, which in turn leads to a better integration of economic and environmental policies.

Furthermore, as argued in both the public economics and environmental literatures (Goulder [1995]), either a tax or a permit policy would generate revenue that could be used as a "double dividend" to not only correct the externality but to also reduce the number of distortions due to taxation of other inputs, such as labor and capital. Moreover, an equivalence between the tax and the permit policies holds when the regulator has symmetric information about all state variables for any outcome under the tax policy and a cap-and-trade scheme (Heutel [2012]).

4 Estimation

In this section, we estimate the structural parameters of the model using Bayesian methods. For a presentation of the method, we refer to the canonical papers of An and Schorfheide [2007] and Smets and Wouters [2007]. Since the U.S. has not implemented any environmental policy, we propose to estimate the *laissez-faire model.* The following sub-sections discuss the non-linear method employed for the estimation, the data transformation as well as the calibration, the priors and the posteriors.

4.1 Solution method

To take into account the effect of risk on asset prices, we employ a tractable likelihood-based method pioneered by Kollmann [2013]. This method, referred to as the inversion filter, allows to perform an estimation of DSGE model up to any order of approximation to the policy rule. In this paper, since we want to accurately measure higher order effects of environmental preferences (e.g. precautionary saving), we consider a second-order approximation to the decision rules of our model. In a nutshell, the inversion filter extracts the sequence of innovations recursively by inverting the observation equation.

One of the drawbacks of this approach lies in the number of shocks that has to be exactly the same as the number of innovations to allow the recursive inversion of the observation equation. Given this limitation, the model is estimated using 4 observable macroeconomic time-series, which are jointly replicated by the model through the joint realization of 4 corresponding innovations. Note that we use the pruning state-space to obtain the matrices of the policy rule using the Dynare package of Adjemian, Bastani, Juillard, Mihoubi, Perendia, Ratto, and Villemot [2011]. From this state-space representation, we reverse the observation equations to obtain the sequence of shocks. Unlike Kollmann [2013] who limits the analysis to a frequentist approach, we augment the likelihood function with prior information in the same spirit as Smets and Wouters [2007]. This method requires a sampler, here Metropolis-Hastings, to get the uncertainty on the estimated value of the model's structural parameters.

4.2 Data

The model is estimated with Bayesian methods on U.S. quarterly data over the sample time period 1973Q1 to 2018Q4, and which are all taken from FRED and the U.S. Energy

Information Administration.

Concerning the transformation of series, the point is to map non-stationary data to a stationary model (namely, GDP, consumption, investment, CO2 emissions). Following Smets and Wouters [2007], data exhibiting a trend or unit root are made stationary in two steps. First, we divide the sample by the working age population. Second, data are taken in logs and we use a first-difference filter to obtain growth rates. Real variables are deflated by the GDP deflator price index. Measurement equations mapping our model to the data are given by:

$$\begin{bmatrix} \text{Real Per Capita Output Growth} \\ \text{Real Per Capita Consumption Growth} \\ \text{Real Per Capita Investment Growth} \\ \text{Per Capita } CO_2 \text{ Emissions Growth} \end{bmatrix} = \begin{bmatrix} \log \gamma_A + \Delta \log \left(\tilde{y}_t\right) \\ \log \gamma_A + \Delta \log \left(\tilde{c}_t\right) \\ \log \gamma_A + \Delta \log \left(\tilde{\imath}_t\right) \\ \log \gamma_A^{1-\varphi_2} \gamma_E + \Delta \log \left(\tilde{e}_t\right) \end{bmatrix}, \tag{25}$$

where a variable with a tilda, $\tilde{x}_t$, denotes the detrended expression of a level variable, x_t.

4.3 Calibration and prior distributions

Calibrated parameters are reported in Table 4. For parameters related to business cycle theory, their calibration is standard: the depreciation rate of physical capital is set at 2.5 percent in quarterly terms, the government spending to GDP ratio to 20 percent and the share of hours worked per day to 20 percent. The environmental component parameters of the models, when not estimated, are set in a similar fashion as Nordhaus [2008] and Heutel [2012]. For parameter φ_1, we set this parameter to match the ratio of CO2 emissions-to-GDP to 1.5558. This ratio corresponds to the average number of kiloton of CO2 emitted per real unit produced in 2019 U.S. dollars in our sample period. The continuation rate of carbon in the atmosphere, denoted η, is set to match a roughly 139 years half time of atmospheric carbon dioxide as in Nordhaus [1991].[10] Finally, for the abatement cost function, we set

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[10] Let us assume that each unit of CO2 is subject to an idiosyncratic shock, denoted ω, that the carbon is reused or sequested in a carbon sink. This random variable is drawn from a binomial distribution, $\omega \sim B(n,p)$ with n the number of trials and p the probability of success $p = 1 - \tilde{\eta}$. We thus determine the number of trials, n, that are necessary on average for one unit of carbon to be sequestrated. Recall that $E\left(\omega\right) = n.p$, by imposing, $E\left(\omega\right) = 1$, we compute that the average number of trials necessary for carbon sequestration is $n = 1/\left(1 - \tilde{\eta}\right)$. In annual basis, the latter becomes $n = 0.25/\left(1 - \tilde{\eta}\right)$. Recall that in the balanced growth path, the effective continuation rate of carbon is $\tilde{\eta} = \eta \gamma_A \gamma_E^{1-\varphi_2}$. Then imposing an average half time of carbon of 139, we deduct the value of η as: $\tilde{\eta} = \left(1 - 0.25/139\right)\left(\gamma_A \gamma_E^{1-\varphi_2}\right)^{-1}$.

$\theta_1 = 0.05607$ and $\theta_2 = 2.8$ as in Nordhaus [2008] and Heutel [2012].

For the remaining set of parameters and shocks, we employ Bayesian methods. Table 5 summarizes the prior — as well as the posterior — distributions of the structural parameters for the U.S. economy. The prior information on the persistence of the Markov processes as well as the standard deviation of innovations are taken from Guerrieri and Iacoviello [2017]. In particular, the persistence of shocks follows a beta distribution with a mean of 0.5 and a standard deviation of 0.2, while for the standard deviation of shocks we choose an inverse gamma distribution with mean 0.01 and standard deviation of 1. For the parameters which have key asset pricing implications, we translate some bound restrictions from the matching moments exercise of Jermann [1998] into prior distributions. In particular, the elasticity of Tobin's Q to the investment-capital ratio is assumed to follow a Gamma distribution with prior mean 4 and standard deviation of 1. The latter implies a support for ϵ close to the bound $\epsilon \in [0.16; +\infty]$ of Jermann [1998]. In addition, we set the capital intensity α to follow a Beta distribution with mean 0.25 and standard deviation 0.02 in order to be close to the value estimated by Jermann [1998]. Note that we set a tight prior on this parameter in order to match the tight interval range of α that replicates the U.S. investment-to-output ratio. For the risk aversion coefficient, Jermann [1998] calibrates its value to 5 to be consistent with asset pricing models. However, a high value for σ typically generates a strong consumption smoothing behavior on the Euler equation that is at odds with the data. Environmental economics typically favors value close to 2, while likelihood-estimated models find values usually below 2 (e.g. Smets and Wouters [2007]). To reconcile these three literatures, we propose to estimate agnostically this key parameter by imposing a rather diffuse information through a Gamma distribution with prior mean of 2 and standard deviation of 0.35. This prior allows the parameter to be either high (i.e. close to 5) as in asset pricing models or lower (i.e. close to 2) as in environmental models as in Stern [2008] and Weitzman [2007], or low (i.e. equal to one) as in estimated business cycle models. Unlike Jermann [1998], we cannot estimate directly $\beta\gamma_A^{-\sigma}$, because of a weak identification when using full-information methods. We thus follow Smets and Wouters [2007] and estimate instead the term $(1/\beta - 1)100$: this allows to easily impose a prior information based on a Gamma distribution with mean 0.5 and standard deviation 0.25. The resulting prior allows the discount factor to roughly lie between 0.99 and 0.9980.[11] The growth rate of productivity (denoted $(\gamma_A - 1) \times 100$) is

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[11] Note in addition that our prior mean for $(1/\beta - 1)100$ is much higher than Smets and Wouters [2007] because our model is non-linear and thus features the precautionary saving effect that drives down the real

given by a Gamma distribution with prior mean of 0.5 and a standard deviation of 0.04 in order to match the average 0.40 percent quarterly growth rate. For the (de)coupling rate (denoted $(\gamma_E - 1) \times 100$), we let the data be fully informative about the slope through a normal distribution with prior mean 0 and standard deviation 0.25. Finally, the last remaining parameter is the utility loss from cumulative CO2 emissions ϕ. As in Campbell and Cochrane [1999], and given that we have several exogenous shocks, this parameter must be restricted to ensure that surplus consumption remains always positive. This restriction ensures non-negativity for the Lagrangian multiplier on the budget constraint (otherwise the budget constraint would not bind). We thus express this parameter in terms of steady state consumption, $\phi\bar{c}/\bar{x}$, and impose an uninformative prior with an uniform distribution with mean 0.5 and standard deviation 0.285. This prior induces a bound restriction such that $\phi\bar{c}/\bar{x} \in [0; 1]$, this is rather conservative because – unlike Beta distributions – it does not favor any value within this interval.[12]

4.4 Posterior distributions

In addition to prior distributions, Table 5 reports the means and the 5th and 95th percentiles of the posterior distributions drawn from two parallel MCMC chains of 35,000 iterations each. The sampler employed to draw the posterior distributions is the Metropolis-Hasting algorithm with jump scale factor so as to match an average acceptance rate close to 25 percent for each chain.

Results of the posterior distributions for each estimated parameter are reported in Table 5 and Figure 1. It is clear from Figure 1 that the data were informative as the shape of the posterior distributions is different from the priors. Our estimates of the structural parameters common with Smets and Wouters [2007] are in line with the estimates of these authors. Indeed, the persistence of productivity and spending shocks are for instance very similar to the canonical paper. In addition, the risk aversion coefficient σ has a posterior mean of 4.2, which is lower than the value of Jermann [1998]. It is however higher than the values reported in environmental macroeconomic and estimated DSGE models. For example, Smets and Wouters [2007] find a value of 1.38 for this parameter. Another key parameter that

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rate. Under the prior information of Smets and Wouters [2007], we would obtain a real rate below zero, we thus re-adjust the prior information to make our non-linear model consistent with US real rate data.

[12] Note that with bounds $\hat{\phi} = \phi\bar{c}/\bar{x} \in [0; 1)$, the MRS$=\bar{c} - \phi\bar{x} = \bar{c} - \hat{\phi}\bar{c}$ as in any standard model featuring external consumption habits.

determines the consumption surplus is $\phi\bar{c}/\bar{x}$. We find a value of 0.67 which is very close to the value estimated by Smets and Wouters [2007] in the case of external consumption habits, i.e. 0.71 . The corresponding value of ϕ, given the steady state ratio $\bar{c}/\bar{x}$, is $4e-04$. Regarding the growth rate of productivity, our estimated value, 0.34, is lower than the value of Smets and Wouters [2007] but this is not surprising because economic growth has been lower in our sample, since we exclude the 60s and also include the last decade. Regarding the last estimated parameter common with Smets and Wouters [2007], the data suggest a value for the capital intensity α close to 0.41, which is higher than the estimated values of Jermann [1998] and Smets and Wouters [2007]. This result is important as estimated DSGE models typically predict very low values for α at odds with data on both the capital structure of firms and the investment-to-output ratio. Finally for the discount rate, denoted $100\left(\beta^{-1}-1\right)$, we find a posterior mean of 0.13 that generates a discount factor of 0.9987.

The last remaining parameters are not common with Smets and Wouters [2007]. For the elasticity of Tobin's Q to the investment capital ratio ϵ, we find a posterior mean of 1.44. Relative to Jermann [1998], this value implies a lower degree of adjustment costs. Regarding the elasticity of emissions to output φ_2, we find a value that is remarkably close to the one estimated by Heutel [2012]. Finally for the decoupling rate, we find that the energy-saving technological change has caused reductions in CO2 by about 2% annually.

	Mean		Stand. Dev		Corr. w/ output	
	Data [5%;95%]	Model	Data [5%;95%]	Model	Data [5%;95%]	Model
$100 \times \Delta \log\left(y_t\right)$	[0.28;0.50]	0.34	[0.69;0.85]	0.81	[1.00;1.00]	1.00
$100 \times \Delta \log\left(c_t\right)$	[0.36;0.55]	0.34	[0.60;0.74]	0.90	[0.54;0.76]	0.58
$100 \times \Delta \log\left(i_t\right)$	[0.07;0.68]	0.34	[1.91;2.34]	2.59	[0.61;0.80]	0.72
$100 \times \Delta \log\left(e_t\right)$	[-0.53;0.07]	-0.26	[1.88;2.31]	2.12	[-0.01;0.35]	0.25

Table 1: Data moments vs. model moments (with parameters taken at their posterior mean)

To assess the relevance of the estimated model, as in Jermann [1998], we compare the observable moments taken at a 90 percent interval versus the asymptotic moments generated by the model using a second-order approximation to the policy function. Table 1 reports the results. We find that our model does a reasonably good job at replicating some salient features of the data, as most of the moments simulated by the estimated model falls within the 95 percent confidence interval of the data.

The advantage of using Bayesian estimation is that the model can replicate the historical path of the observable variables that we introduce. Once the shock process parameters are estimated, it is then possible to simulate the model by drawing shocks from the estimated distribution. As illustrated in Table 1, however, this procedure does not ensure that the unconditional standard deviations observed in the data can be matched perfectly.

	Standard Cons. habits	Pollution externality
Utility function $u(c_t - \mathcal{C}_t)$	$\mathcal{C}_t = \phi c_{t-1}$	$\mathcal{C}_t = \phi_t x_t$
Surplus ratio parameter ϕ	0.99	0.67
Prior probability	0.50	0.50
Log marginal data density	1992.10	2045.99
Bayes ratio	1.0000000000	2.53650205243e23
Posterior model probability	0.0000000000	1.0000000000

Table 2: Prior and posterior model probabilities comparison between the internal consumption habits model vs. environmental preferences model (with parameters taken at their posterior mode).

Letting $u(c_t - \mathcal{C}_t)$ denote the utility function with $\mathcal{C}_t$ the reference variable to compute the surplus consumption ratio, a natural question to ask at this stage is how relevant is our specification of environmental preferences with respect to a standard consumption habits model *à la* Jermann [1998]. Using an uninformative prior distribution over models (i.e. 50% prior probability for each model), we compute in Table 2 both posterior odds ratios and model probabilities taking the consumption habits model $\mathcal{M}\,(\mathcal{C}_t = \phi c_{t-1})$ as the benchmark model. We examine the hypothesis H_0: $\mathcal{C}_t = \phi c_{t-1}$ against the hypothesis H_1: $\mathcal{C}_t = \phi_t x_t$. The posterior odds of the null hypothesis of surplus based on lagged consumption is 2.5e23:1 which leads us to strongly reject the null. The surplus consumption ratio is therefore more relevant when it is based on the stock of emissions rather than past consumption. This result must however be qualified as prior distributions were selected here to estimate our model and do not necessarily fit the benchmark model of H_0. This can deteriorate the empirical performance of the benchmark. The goal of this exercise is not to show that one model outperforms another, but to highlight that our model is least as consistent with the data as the standard habit-type model.

5 Results

Our main simulation results are shown in Table 3 below. The upper part of this table reports the average level of consumption as well as the stock of CO2 emissions, which are denoted by $E(c_t)$ and $E(x_t)$, respectively. The agent's lifetime utility, which is denoted by $E(\mathcal{W}_t)$, is our measure of welfare. The average tax chosen by the social planner is denoted by $E(\tau_t)$.

The asset pricing implications are reported in the middle part where $400E(r_t^F)$, $400E\left(r_{t+1}^B - r_t^F\right)$ and $std(\hat{\lambda}_t)$ denote the mean real risk-free rate, the mean bond premium, expressed in annualized percent, and the standard deviation of marginal utility, respectively. The average coefficient of relative risk aversion is denoted by $E(RRA_t)$ whereas $std(\widehat{rra}_t)$ is a measure of its standard deviation expressed in log-deviation from steady state.

The lower part of Table 3 firstly reports the share of emissions that firms choose to abate $E(\mu_t)$. The average cost of abatement is denoted by $E(f(\mu_t))$, and $E(\tau_t e_t / y_t)$ is the average cost of the tax borne by firms as a share of GDP.

The first column shows these model implications in the *laissez-faire* equilibrium, which corresponds to the decentralized equilibrium with a tax set to zero. Columns (2) to (4) show what happens once the optimal tax is introduced. Under the optimal policy, the results are reported for three different values for the parameter θ_1. The latter measures the efficiency of the abatement technology, where an increase in θ_1 corresponds to the case of a less efficient technology. Since $\theta_1 = 0.05607$ is the value used in the literature (e.g. Nordhaus [2008]; Heutel [2012]), the results reported in column (2) correspond to our baseline scenario.

5.1 The size and the cyclicality of the optimal tax

The first main takeaway from Table 3 is that a small average carbon tax is sufficient to restore the first-best allocation. Indeed, under our benchmark scenario, which corresponds to the case $\theta_1 = 0.05607$, it optimal to impose an average tax of around 2.1 percent. As can be seen by comparing the level of the tax across columns 2 to 4, in the worst case scenario, the average tax only reaches around 2.5 percent. This worse case scenario corresponds to a value for θ_1 implying that the abatement technology available in the economy is highly inefficient. Under such an adverse scenario, firms only manage to abate about 5.6 percent of all emissions, i.e. $E(\mu_t) = 0.0562$, once the tax is introduced.

One advantage of our methodology is that it can be used to construct counterfactual scenarios. In particular, we can answer the following question: what would have been the level of the optimal tax in the United States from 1973 to 2018, had this optimal policy been implemented. Figure 2 provides the answer. The optimal tax is time-varying. It should increase in boom times and decline during recessions.

The intuition for this result is that the externality induces fluctuations in risk aversion that are excessive. As in a model with external habits and time-varying risk aversion (e.g. Campbell and Cochrane [1999]), agents take the externality as given. Since the optimal tax reproduces the first-best allocation, it eliminates this inefficiency by making firms internalize the effect of their production on consumers. Our analysis therefore provides a novel interpretation to the result obtained by Ljungqvist and Uhlig [2000] in the case of habits. As shown in Table 2, one motivation for our approach is that our specification is strongly supported by the data, especially relative to habits.

It is important to note that the fluctuations in risk aversion are essentially driven by consumption, not the externality. In line with the evidence, we assume that the stock of CO2 depreciates very slowly over time. Whereas the flow of emissions can be volatile, the stock of emissions, and hence the externality, moves very slowly over the business cycle.

5.2 The risk premium and the risk-free rate in the laissez-faire equilibrium

As can be seen in column (1), the model generates an average bond premium, i.e. $400E\left(r^B_{t+1} - r^F_t\right)$, of about 1%. Although small, generating a bond premium of this magnitude remains a challenge for a large class of general equilibrium models with production. In our case, this relative success is due to our preference specification, which generates time-variation in risk aversion as in Campbell and Cochrane [1999].

As in Jermann [1998], the positive bond premium that we obtain is due to interest rate risk. Indeed, the price of a long-term bond is determined by the term structure of interest rates. The key is that in this model short- and long-term interest rates are countercyclical. Since interest rates rise during recessions, bond holders can expect capital losses to occur precisely during periods of low consumption and high marginal utility. Long-term bonds are therefore not good hedges against consumption risk. Consequently, the positive bond premium is a compensation for holding an asset whose price declines during periods of low consumption.

In this model, the mean risk-free rate $400E(r^F_t)$ is critically affected by uncertainty.

Indeed, as in Jermann [1998], an increase in the variance of marginal utility reduces the unconditional mean risk-free rate. The intuition is that a higher volatility of marginal utility implies more uncertainty about future valuations. The key is that a rise in uncertainty in turn increases agents' willingness to build precautionary buffers. This effect therefore captures the impact of this precautionary motive on equilibrium interest rates.

5.3 Asset prices under the optimal policy

Relative to the *laissez-faire* equilibrium, the optimal tax has a sizeable effect on the mean risk-free rate. In the baseline scenario, under optimal taxation, our model predicts an increase in the average risk-free rate of almost 2 percent. This effect on the risk-free rate can be better understood by comparing the volatility of marginal utility $std(\hat{\lambda}_t)$ in the two cases. A main effect of the tax is to reduce the volatility of marginal utility. Fluctuations in marginal utility provide a measure of uncertainty about future valuations. This decline in volatility therefore illustrates that agents face less uncertainty once the tax is introduced. The increase in the mean risk-free rate can therefore be interpreted as a reduction in agents' precautionary saving motives.

The second effect of the tax is to decrease the risk premium. This result can be explained by the effect of the tax on risk aversion. The carbon tax reduces both consumption and the stock of emissions. The key is that the tax causes a reduction in the stock of emissions that exceeds the decline in consumption. The distance between consumption and the externality therefore increases. In this model, a higher gap between consumption and the externality in turn lowers risk aversion.

In contrast to an endowment economy, in our production economy, a decline in risk aversion also affects the dynamics of consumption. In this environment, a lower risk aversion implies a higher elasticity of intertemporal substitution (EIS). In other words, the agent's consumption smoothing motive becomes weaker under the optimal policy. This willingness to tolerate larger fluctuations in consumption in turn has asset pricing implications. Indeed, since agents are less reluctant to reduce consumption during recessions, the need to insure against such outcomes is less pressing. Consequently, the premium needed to compensate investors for holding an asset whose price declines in recessions is also lower.

5.4 *Welfare analysis*

To assess the welfare implications of the optimal policy, Table 3 also reports agents' lifetime utility $E\left(\mathcal{W}_t\right)$, where:

$$E(\mathcal{W}_t) = E_0\left\{\sum_{t=0}^{\infty}\beta^t\frac{\left(c_t - \phi_t x_t\right)^{1-\sigma}}{1-\sigma}\right\}$$

As can be seen by comparing the value of $E\left(\mathcal{W}_t\right)$ across columns (1) and (2) the policy generates a sizeable increase in welfare. This welfare gain illustrates that the decline in the stock of emissions $E(x_t)$ more than compensates for the decline in average consumption induced by the tax. This result highlights the importance of the elasticity of emissions to a change in the tax. Since this elasticity in turn depends on firms' willingness to reduce emissions, we next discuss the role of the abatement technology.

5.5 *The role of the abatement technology*

The purpose of columns (3) and (4) is to illustrate that the effect of the optimal tax critically depends on the efficiency of the abatement technology. In the laissez-faire equilibrium, the fact that the externality is not internalized leads firms to choose zero abatement expenditures. By forcing firms to internalize the effect of the externality, one key effect of the tax is to incentivize firms to use the abatement technology to reduce the burden of the tax.

In our preferred scenario, about 75 percent of emissions are abated once the optimal tax is introduced. As shown in the lower portion of Table 3, as θ_1 rises above 0.056, a reduction in the efficiency of the technology reduces the share of emissions abated $E\left(\mu_t\right)$. Note that when the efficiency of the abatement technology declines, the planner also chooses to allocate a larger fraction of resources to consumption. This reflects that this model embeds a trade-off between consumption and the abatement technology. The marginal cost of renouncing to one unit of consumption has to be equal to the marginal benefit from abating one unit of emission. Consequently, the planner finds it optimal to allocate more resources to consumption when the efficiency of the abatement technology declines.

As can be seen by comparing $E\left(\mathcal{W}_t\right)$ across columns (2) to (4), the magnitude of the welfare gain critically depends on the abatement technology. This illustrates that the distortion caused by the tax can be sizeable if the technology is not sufficiently well-developed. Indeed,

	Laissez-faire	Optimal policy		
	Estimation (1972-2019)	$\theta_1 = 0.05607$	$\theta_1 = 0.56797$	$\theta_1 = 6.8844$
	(1)	(2)	(3)	(4)
Business cycle variables				
$E\left(c_t\right)$	0.5484	0.5014	0.5268	0.5398
$E\left(x_t\right)$	932.0311	222.3127	702.9981	860.9645
$E(W_t)$	-18694.3	-515.3	-4306.3	-11739.6
$E(\tau_t)$	0.0000	0.0213	0.0235	0.0253
Asset pricing implications				
$400E\left(r_t^F\right)$	3.6118	5.6544	4.7403	4.0130
$400E\left(r_{t+1}^B - r_t^F\right)$	1.1052	0.4818	0.8813	1.1071
$std(\hat{\lambda}_t)$	2.4673	1.0990	1.7941	2.2308
$E(RRA_t)$	32.1324	10.8755	20.2128	27.5478
$std(\widehat{rra}_t)$	0.5876	0.2617	0.4273	0.5312
Abatement technology				
$E\left(\mu_t\right)$	0.0000	0.7464	0.2162	0.0562
$E\left(f\left(\mu_t\right)\right)$	0.0000	0.0250	0.0079	0.0022
$E(\frac{\tau_t e_t}{y_t})$	0.0000	0.0081	0.0289	0.0374

Notes: The first column is the estimated model under a laissez-faire equilibrium, with no abatement and no environmental tax. Column (2) is the equilibrium under an environmental tax with θ_1 set as in the litterature. Columns (3) and (4) are equilibriums under alternative values of θ_1 that matches a share of abattement $\bar{\mu}$ of 20% and 5%. Note that in columns (3) and (4) $E(\mu_t) \neq \bar{\mu}$ because of the contribution of futur shocks on the asymptotic mean of these variables.

Table 3: In column (1), the model simulations correspond to the laissez-faire equilibrium. The simulations under the optimal environmental policy are shown in columns (2) to (4). Columns (2) to (4) correspond to different abatement costs, ranging from low to high.

if emissions are costly to abate, the policy has a stronger negative impact on production, as it is more difficult for firms to circumvent the tax. In this case, the tax generates a smaller decline in emissions, which in turn reduces the welfare gains from the policy.

As can be seen by comparing the effect of the optimal tax on $400E\left(r_t^F\right)$ and $400E\left(r_{t+1}^B - r_t^F\right)$, the effect on asset prices also crucially depends on the parameter θ_1. Relative to the first-best scenario, the effect of the tax on the risk premium is more muted when the abatement technology is less efficient. This illustrates that part of the reduction in uncertainty is due to the additional margin provided by the abatement technology. The effect of the parameter θ_1 is therefore akin to the adjustment cost parameter in Jermann [1998]. The more efficient the abatement technology, the easier it is for agents to insure against unexpected shocks. This higher flexibility makes the economy less risky from a consumption smoothing perspective, an effect which reduces the risk premium and increases the risk-free rate.

5.6 *Coefficient of relative risk aversion*

Table 3 also reports the average level of risk aversion, where risk aversion is defined as follows:

$$RRA_t = -\frac{u''_c}{u'_c} c_t$$

In the *laissez-faire* equilibrium, the average level of risk aversion is 32. Interestingly, once the tax is introduced, the coefficient of risk aversion declines to around 11. This illustrates that the main effect of the tax is to increase the distance between consumption and the externality. As in Campbell and Cochrane [1999], in our model, risk aversion is determined by "surplus consumption". A higher distance between consumption and the externality therefore implies a lower coefficient of relative risk aversion.

The efficiency of the abatement technology has a major impact on the effect of the tax on the coefficient of relative risk aversion $E(RRA_t)$. Even when the abatement technology is less efficient, relative to the *laissez-faire* equilibrium, the planner still finds it optimal to reduce the level of consumption. However, since it is harder to reduce the stock of emissions when the abatement technology is less efficient, the effect on the externality is more muted. As a result, relative to the baseline scenario reported in column 2, the tax leads to a smaller increase in "surplus consumption". As can be seen by comparing columns (3) and (4) to column (2), risk aversion under the optimal policy is therefore higher when the technology available in the economy is not sufficiently well-developed.

5.7 *The responses to shocks*

Figure 3 below compares the response of consumption c, the abatement μ, emissions e and the optimal tax τ in the case of a positive technology shock. As can be seen by comparing the red crossed line with the green circles in the upper left panel, on impact, the response of consumption is stronger under the optimal policy. This effect can be explained by the lower EIS. In models with habits, relative risk aversion and the EIS are connected. Since the tax reduces risk aversion, it also increases the EIS.

As illustrated by the upper right panel of Figure 3, the second key difference is that the quantity of emissions that firms choose to abate increases sharply during boom periods. Once firms are forced to internalize the effects of their production on consumers, it becomes optimal to use the abatement technology.

As shown by the lower left panel of Figure 3, the procyclical response of the abatement technology implies a decline in emissions under the optimal policy. In contrast to the *laissez-faire* equilibrium, emissions therefore become counter-cyclical once the optimal tax is introduced.

Finally, the lower right panel of Figure 3 shows the response of the optimal tax, which is constant and equal to zero in the *laissez-faire* equilibrium. As in Ljungqvist and Uhlig [2000], we find that the optimal tax is pro-cyclical when the economy is hit by a technology shock. Relative to the decentralized equilibrium, the planner therefore chooses to cool down the economy during periods of booms.

The investment-specific technology shock has an impact on the environmental variables that is qualitatively similar to that of a technology shock (see Figure 4). The key difference is that the investment-specific shock generates a negative co-movement between consumption and investment. Relative to Jermann [1998], introducing this shock reduces the volatility of investment, which in turn explains the lower value for the adjustment cost parameter that we obtain.

The response to a government spending shock is shown in Figure 5. In both cases, notice that a positive government spending shock reduces consumption. In this model, this can firstly be explained by the negative wealth effect induced by the shock. On impact, the shock has no effect on production but increases the share of output allocated to government spending. On impact, consumption and investment therefore have to fall.

The negative wealth effect is reinforced by a negative substitution effect. As in models with habits and adjustment costs, this is due to the increase in the real interest rate generated by the shock. This reflects that agents become more reluctant to save when consumption falls. Consequently, the real interest rate has to increase to equilibrate the market.

This case illustrates the trade-off between environmental protection and macroeconomic stabilization present in this model. Whereas emissions decline in the *laissez-faire* case, the social planner chooses to increase the stock of pollution. The social planner internalizes that the shock reduces the resources available for consumption. It is therefore optimal to mitigate the effect of the shock by lowering the abatement effort as well as the tax (see upper right and lower right panels of Figure 5). When the consumption cost is too large, the environmental policy is used to mitigate the adverse effect of the shock. In this case, the planner therefore chooses macroeconomic stabilization over environmental protection.

Relative to a standard business cycle model, the introduction of emission shocks is the main innovation. In the *laissez-faire* equilibrium, consumption declines on impact and then increases above steady state (see upper left panel of Figure 6). Since an emission shock does not affect output, its main effect is to reduce "surplus consumption". As a result, the only way to mitigate the effect of this increase in the stock of emissions is to increase consumption. The problem is that income needs to increase first. In this model, the only way to raise income is to accumulate capital. This explains why on impact consumption needs to fall. This fall is necessary to finance an increase in investment, which in turn allows agents to increase output. A few quarters after the shock, as the increase in investment raises output, consumption gradually increases. The short-term decline in consumption is therefore compensated by an increase in the medium-term. As illustrated by the red dotted line in the upper left panel of Figure 6, this explains why consumption initially declines and then increases above steady state a few periods after the shock.

As can be seen by comparing the red dotted lines with the green circled line, the response of consumption and emissions is very different under the optimal policy. The planner chooses to allocate a large fraction of resources to the abatement technology. It is therefore optimal to reduce consumption and investment to finance the abatement, which then leads to a reduction in emissions.

As illustrated by the lower right panel, the social planner also chooses to implement a small reduction in the tax. The tax reduction helps to mitigate the fall in consumption and investment that is necessary to finance the abatement effort.

5.8 The relationship between the externality and risk premiums

Turning now to the risk premium and the externality parameter ϕ, as illustrated in the left panel of Figure 7, the risk premium increases with ϕ in the *laissez-faire* equilibrium. As previously highlighted, risk aversion depends on the distance between consumption and the stock of emissions. As a result, a higher marginal loss of emissions reduces the distance between consumption and the externality. Since a smaller surplus in turn implies a greater coefficient of relative risk aversion, we obtain a positive relationship between ϕ and the risk premium in the *laissez-faire* equilibrium. In other words, when firms fail to internalize the damage caused by their emissions, risk aversion increases. Consequently, the risk premium demanded by investors rises. As shown by the right panel of Figure 7, increasing the externality parameter in turn raises to social cost of the externality.

Under the optimal policy, a disconnect between the externality parameter and the risk premium emerges. As illustrated by the blue continuous line in the left panel of Figure 8, increasing ϕ beyond its estimated value reduces the risk premium in our baseline scenario. Relative to the *laissez-faire* case, the key is that the social planner optimally chooses not only consumption but also the level of pollution x. In particular, if the technology is sufficiently efficient, the tax can be used to achieve large reductions in the stock of pollution. Consequently, an increase in ϕ does not necessarily lower the surplus, and hence raises risk aversion, if the tax can be used simultaneously to achieve large reductions in x.

Under the optimal policy, the effect of the marginal damage parameter on the tax as well as the risk premium is illustrated in the right panel of Figure 8. The blue continuous line denotes the benchmark scenario, which corresponds to the case of an efficient technology. In this case, the key is that the tax is sensitive to the value of ϕ. Since firms have access to an efficient technology, a small increase in the tax is sufficient to induce a large quantity of emission abatement.

The decline in the risk premium observed for values of ϕ higher than 0.4 can then be explained by two effects. First, the increase in ϕ only has a small effect on risk aversion when accompanied by a large decline in the stock of pollution. Second, the abatement technology opens a new adjustment margin that the planner can exploit to facilitate consumption smoothing.

To illustrate the importance of emissions abatement in generating this result, the red dotted line shows the case in which the technology is set to an inefficient level. When the cost of abating emissions is too high, while available, this margin of adjustment is not sufficiently flexible to reduce consumption risk. In this case, without a strong reduction in x, an increase in ϕ raises the coefficient of relative risk aversion. Consequently, the relationship between the damage parameter, risk aversion and the risk premium is similar to that obtained in the *laissez-faire* equilibrium.

The main takeaway is therefore that a higher value for the externality parameter increases risk premia in the *laissez-faire* equilibrium. However, under the optimal policy, this relationship is more ambiguous. Whether the externality increases or reduces the risk premium critically depends on the efficiency of the emission abatement technology.

5.9 The cost of abating emissions

In Table 3, the cost of abating emission is reported as a fraction of output. In the *laissez-faire* equilibrium, the abatement technology is not used and the cost is therefore zero. Under the optimal policy, our benchmark scenario corresponds to the case $\theta_1 = 0.056$. This calibration implies that the total abatement cost represents about 2.5 percent of GDP, which is a lower estimate than the one provided in the public economics literature. If the cost of abating emissions represents 2.5 percent of output, *i.e.* $E(f(\mu_t)) = 0.025$, as shown by the corresponding value for $E(\mu_t)$, the social planner chooses to abate around 75 percent of all emissions.

Relative to our benchmark scenario, increasing θ_1 to 0.57 and 6.88 reduces the efficiency of the abatement technology. In this case, the social planner finds it optimal to only abate 21.6 and 5.6 percent of all emissions, respectively. Relative to our benchmark scenario, reducing the efficiency of the abatement by increasing θ_1 raises the cost of the tax that needs to be paid by firms. If the technology is well-developed, which is our benchmark scenario, the cost of the tax represents on average around 0.8 percent of GDP, *i.e.* $E(\tau_t e_t / y_t) = 0.08$. Reducing the efficiency of the technology by setting θ_1 to 0.57 and 6.9, increases the cost of the tax from 0.8 percent to 2.9 percent and 3.7 percent of GDP, respectively.

Relative to the benchmark scenario, the total abatement cost declines from 2.5 percent of GDP to 0.8 and 0.2 percent of GDP, respectively. Since raising θ_1 reduces the efficiency of the technology, firms choose to compensate by substantially lowering the fraction of emissions abated μ. Consequently, whereas the technology is less efficient, the total cost of abatement declines because of the much smaller fraction of emissions that firms choose to abate.

6 Conclusion

Drawing from the macroeconomic, financial, and environmental literatures, this paper introduces an environmental externality into the neoclassical growth model. Our first main takeaway is that the optimal carbon tax is determined by the implicit price of CO2 emissions. We then show how to use asset pricing theory to estimate the optimal carbon tax over the business cycle.

In our economy, risk aversion is higher when firms fail to internalize the damage caused by emissions. We show that this increase in risk aversion in turn raises risk premia and lowers the natural rate of interest by increasing precautionary saving. In the *laissez-faire* equilibrium, the key is that a fraction of these variations in risk aversion are excessive. One interpretation of the optimal policy is therefore that it eliminates the fluctuations in risk aversion that are inefficient.

In terms of policy implications, the main takeaway is that the effectiveness of the policy critically depends on the abatement technology. The success of the policy may therefore depend on the timing of implementation. Clearly, improving the existing emission abatement technology should come first. Once available, an efficient technology would help to mitigate the side effects of the tax, thereby maximizing the welfare gains from the policy.

As our study focuses primarily on tax policy, future research could investigate how a permits market could impact asset prices and welfare, by either considering the case of asymmetric information[13], or by developing a framework where both households and firms are affected by the externality. Such a framework would allow for multi-policy evaluation, such as a comparison between tax and cap and trade policies.

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[13] Asymmetric information breaks the equivalence between the tax and the permit policy (Heutel [2012]).

7 Bibliography

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8 Appendix - A: tables

Model counterpart	Name	Values
$\bar{N}$	Labor supply	0.20
δ_K	Depreciation rate of capital	0.025
$\bar{g}/\bar{y}$	Public spending share in output	0.20
$\bar{e}/\bar{y}$	Emissions-to-output ratio (kilon per 2019 dollar)	1.5558
$[4(1-\gamma_A\gamma_E^{1-\varphi_2}\eta)]^{-1}$	Half-life of CO2 in years	139
θ_1	Abatement cost	0.05607
θ_2	Curvature abattement cost	2.8

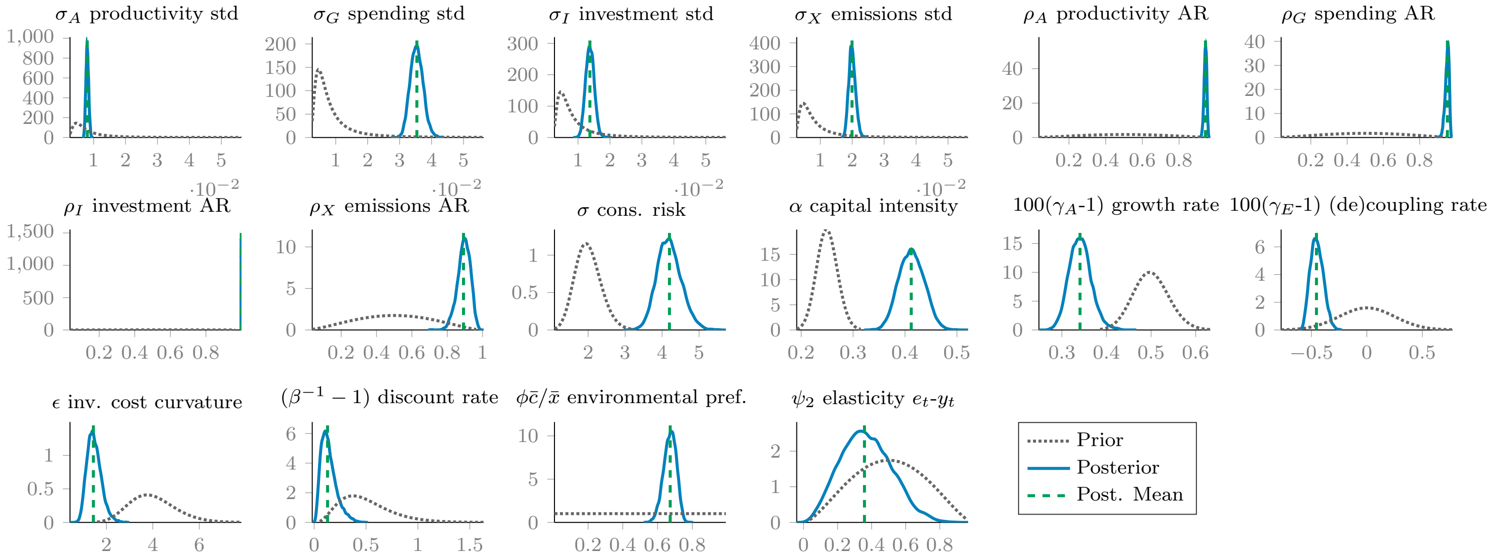
Table 4: Calibrated parameter values (quarterly basis)

		Prior distributions			Posterior distributions
		Shape	Mean	Std.	Mean *[0.050;0.950]*
Shock processes:					
Std. productivity	σ_A	$\mathcal{IG}_1$	0.01	1	0.008 *[0.007;0.009]*
Std. spending	σ_G	$\mathcal{IG}_1$	0.01	1	0.035 *[0.032;0.039]*
Std. abatement	σ_X	$\mathcal{IG}_1$	0.01	1	0.020 *[0.018;0.022]*
Std. investment	σ_I	$\mathcal{IG}_1$	0.01	1	0.014 *[0.012;0.016]*
AR(1) productivity	ρ_A	$\mathcal{B}$	0.50	0.20	0.944 *[0.931;0.954]*
AR(1) spending	ρ_G	$\mathcal{B}$	0.50	0.20	0.953 *[0.931;0.967]*
AR(1) abatement	ρ_X	$\mathcal{B}$	0.50	0.20	0.892 *[0.826;0.945]*
AR(1) investment	ρ_I	$\mathcal{B}$	0.50	0.20	0.998 *[0.998;0.999]*
Structural parameters:					
Productivity growth rate	$(\gamma_A-1)\times 100$	$\mathcal{G}$	0.50	0.04	0.341 *[0.303;0.382]*
Output-CO2 (de)coupling rate	$(\gamma_E-1)\times 100$	$\mathcal{N}$	0	0.25	-0.45 *[-0.54;-0.35]*
Discount rate	$(\beta^{-1}-1)\times 100$	$\mathcal{G}$	0.50	0.25	0.129 *[0.047;0.278]*
Capital intensity	α	$\mathcal{B}$	0.25	0.02	0.412 *[0.373;0.453]*
Capital cost elasticity	ϵ	$\mathcal{G}$	4	1	1.443 *[1.008;2.010]*
Utility loss on emissions	$\phi\times\bar{c}/\bar{x}$	$\mathcal{U}$	0.50	0.285	0.673 *[0.607;0.725]*
Relative risk aversion	σ	$\mathcal{G}$	2.00	0.35	4.199 *[3.675;4.781]*
Output-CO2 elasticity	φ_2	$\mathcal{B}$	0.50	0.20	0.358 *[0.132;0.626]*
Log-marginal data density					2124.69

Notes: $\mathcal{U}$ denotes the Beta distribution, $\mathcal{IG}_1$ the Inverse Gamma (type 1), $\mathcal{N}$ the Normal and $\mathcal{U}$ the uniform one.

Table 5: Prior and Posterior distributions of structural parameters

9 Appendix - B: figures


σ_A productivity std

σ_G spending std

σ_I investment std

σ_X emissions std

ρ_A productivity AR

ρ_G spending AR

ρ_I investment AR

ρ_X emissions AR

σ cons. risk

α capital intensity

$100(\gamma_A$-1) growth rate

$100(\gamma_E$-1) (de)coupling rate

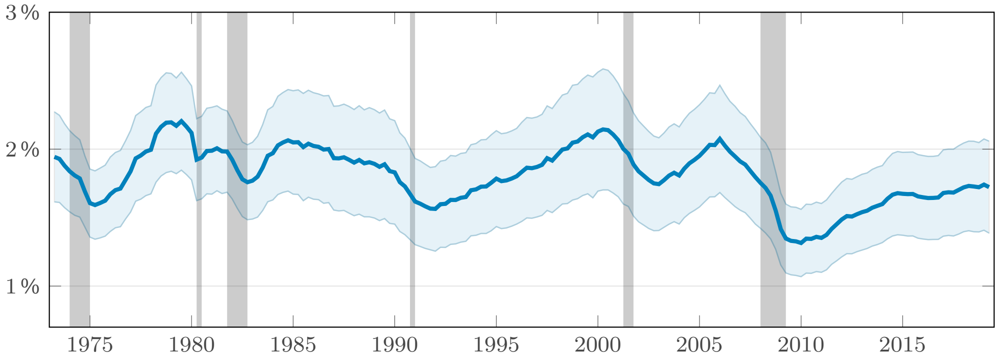
ϵ inv. cost curvature

$(\beta^{-1}-1)$ discount rate

$\phi\bar{c}/\bar{x}$ environmental pref.

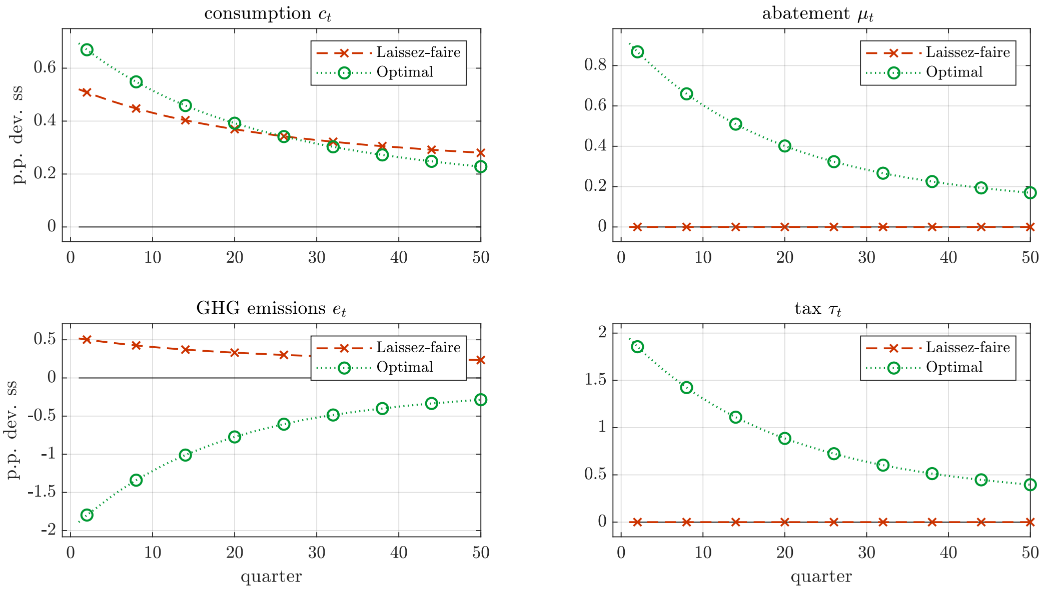
ψ_2 elasticity e_t-y_t

Figure 1: Prior and posterior distributions of estimated parameters



Notes: The simulated path is expressed in percentage deviations from the steady state. The blue shaded area is the parametric uncertainty at 95% confidence level, drawn from 1,000 Metropolis-Hastings random iterations. The blue line represents the mean of these 1,000 simulated paths. The gray shaded areas are NBER-dated recessions in the US.

Figure 2: Historical variations of the environmental tax



Notes: The IRFs are generated using a second order approximation to the policy function and are expressed as percentage deviations from the deterministic steady state. Estimated parameters are taken at their posterior mean.

Figure 3: Impulse responses to an estimated TFP shock

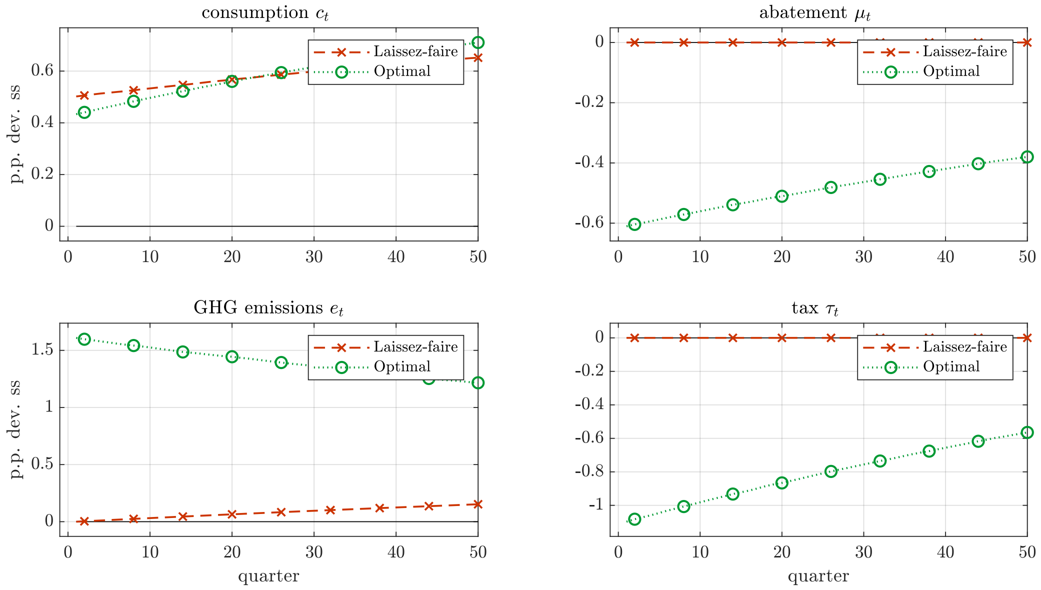


Figure 4: Impulse responses to an investment-specific technology shock

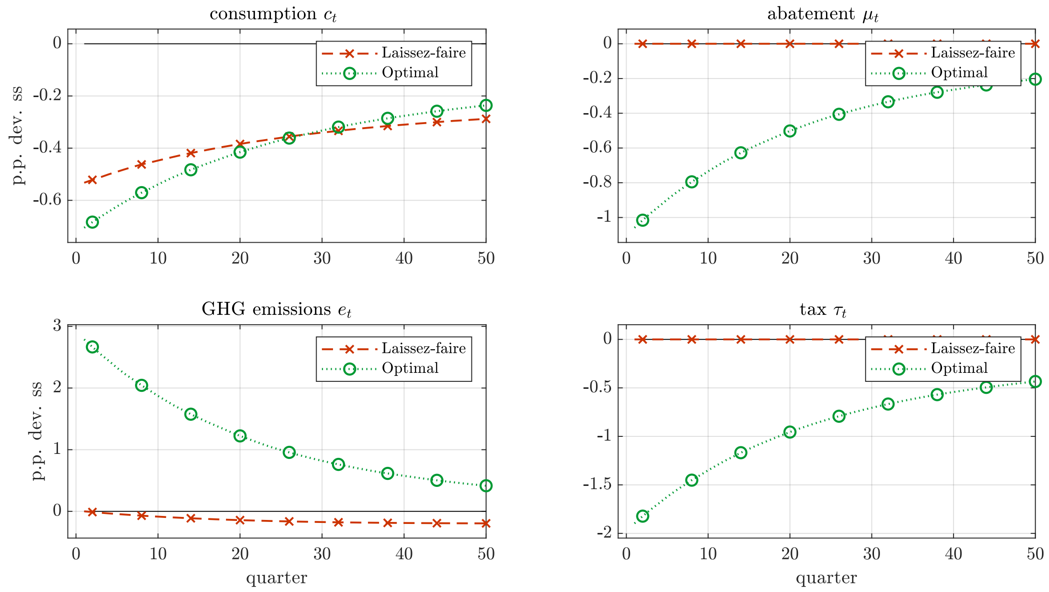


Figure 5: Impulse responses to a government spending shock

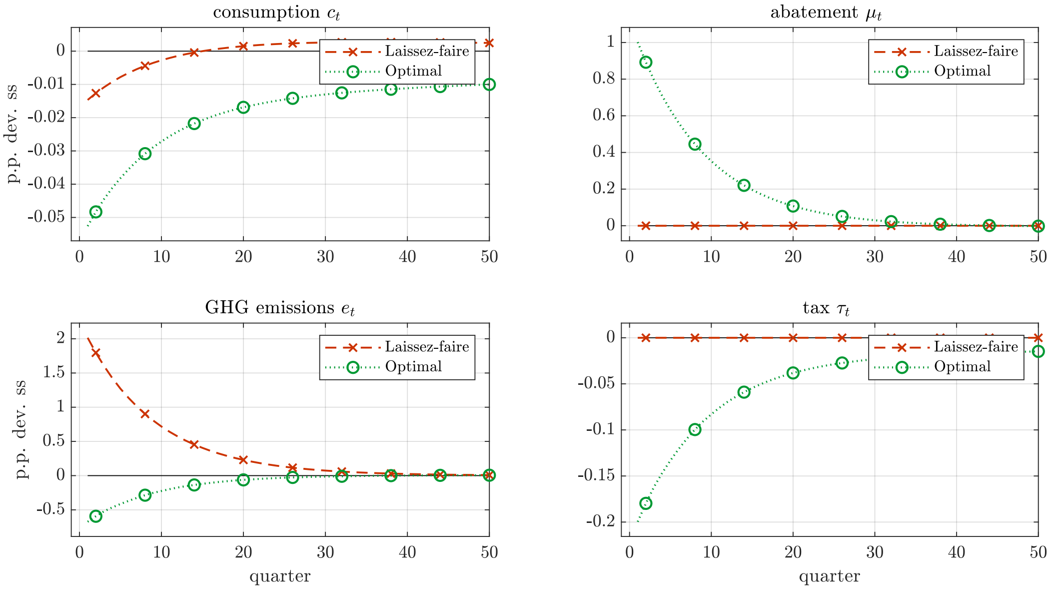
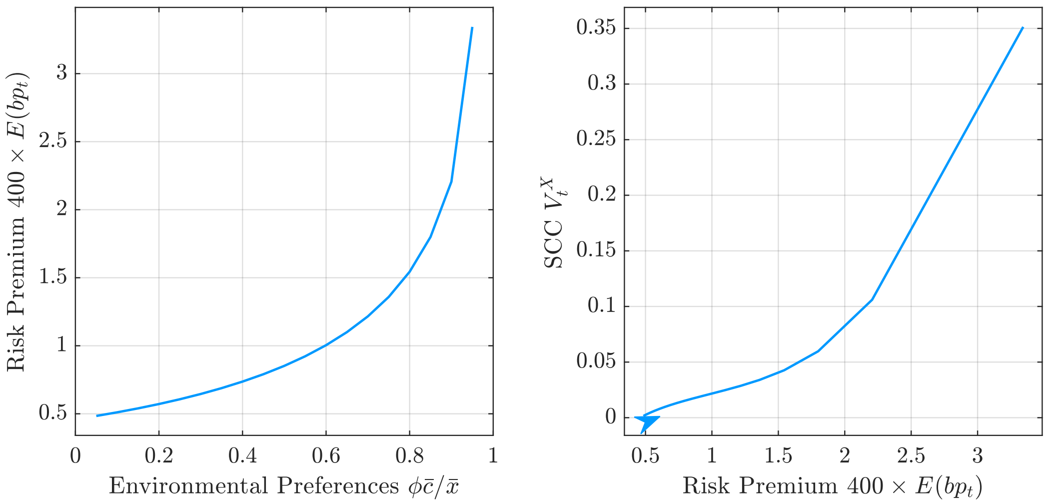
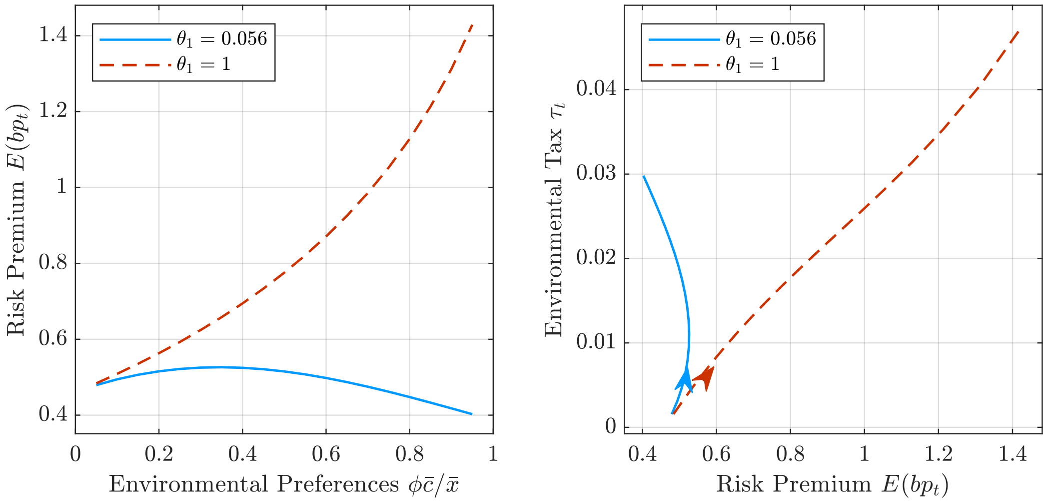


Figure 6: Impulse responses to an emission shock



Notes: The graph on the left reports the interaction between risk premia and environmental preferences in the *laissez-faire equilibrium*. Note that the environmental preference parameter is expressed in steady state consumption to ensure the marginal utility of consumption always remains positive. The right graph reports how environmental preferences shape the social cost of carbon.

Figure 7: Risk premia, preferences, and the social cost of carbon (in the *laissez-faire* equilibrium)



Notes: The graph on the left reports the interaction between risk premia and environmental preferences under two technological values for abatement costs θ_1: cheap abatement (0.056) is plain blue while costly abatement (1) is red dashed. Note that the environmental preference parameter is expressed in steady state consumption to ensure the marginal utility of consumption always remains positive. The right graph reports the interaction between the risk premium and environmental tax for different levels of environmental preferences. The arrow shows the direction when the environmental preferences parameter in the utility function increases from 0.05 to 0.95.

Figure 8: Risk premia, preferences, and tax interactions (under environmental tax policy)

10 Appendix - C: The optimal tax

10.1 Centralized problem

We characterize here the first-best equilibrium. A social planner maximizes welfare which leads producers to internalize the social cost of emissions. The problem for the social planner reads as follows:

$$\begin{aligned}\max E_0 \sum_{t=0}^{\infty} \beta^t \Bigg(& \frac{\left(c_t - \phi_t x_t\right)^{1-\sigma}}{1-\sigma} \\ & + \lambda_t \left[y_t - c_t - i_t - g_t - f\left(\mu_t\right) y_t\right] \\ & + \lambda_t q_t \left[(1-\delta)k_t + \left[\frac{b_1}{1-\epsilon}\left(\varepsilon_t^I \frac{i_t}{k_t}\right)^{1-\epsilon} + b_2\right] k_t - k_{t+1}\right] \\ & + \lambda_t \varrho_t \left[\varepsilon_t^A k_t^{\alpha} \left(\Gamma_t N_t\right)^{1-\alpha} - y_t\right] \\ & + \lambda_t V_t^X \left[x_{t+1} - \eta x_t - e_t\right] \\ & + \lambda_t V_t^E \left[e_t - \left(1-\mu_t\right) \varepsilon_t^X \varphi_1 y_t^{1-\varphi_2} \Psi_t\right] \Bigg)\end{aligned}$$

First order conditions are given by:

$$c_t : \lambda_t = \left(c_t - \phi_t x_t\right)^{-\sigma}$$

$$i_t : 1 = \varepsilon_t^I q_t b_1 \left(\varepsilon_t^I \frac{i_t}{k_t}\right)^{-\epsilon}$$

$$k_{t+1} : q_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ q_{t+1} \left((1-\delta_K) + \frac{b_1}{1-\epsilon}\left(\varepsilon_{t+1}^I \frac{i_{t+1}}{k_{t+1}}\right)^{1-\epsilon} + b_2 - b_1 \left(\varepsilon_{t+1}^I \frac{i_{t+1}}{k_{t+1}}\right)^{1-\epsilon} \right) + \varrho_{t+1} \alpha \frac{y_{t+1}}{k_{t+1}} \right\}$$

$$y_t : \left[1 - f\left(\mu_t\right)\right] - \varrho_t - V_t^E \left(1-\varphi_2\right) \frac{e_t}{y_t} = 0$$

$$\mu_t : f'\left(\mu_t\right) y_t = V_t^E \frac{e_t}{\left(1-\mu_t\right)} = 0 \qquad (26)$$

$$e_t : V_t^E = V_t^X \tag{27}$$

$$\lambda_t V_t^X = \beta E_t \phi_{t+1} \left(c_{t+1} - \phi_{t+1} x_{t+1}\right)^{-\sigma} + \eta \beta E_t \lambda_{t+1} V_{t+1}^X \tag{28}$$

10.2 Laissez-faire equilibrium

Firms face the following optimization problem:

$$\begin{aligned} &\max_{\{y_t, N_t, k_t, \mu_t, e_t\}} y_t - w_t N_t - r_t k_t - f\left(\mu_t\right) y_t \\ &+ \varrho_t \left[A_t k_t^\alpha \left(\Gamma_t N_t\right)^{1-\alpha} - y_t\right] \\ &+ J_t^E \left[e_t - \left(1-\mu_t\right) \varepsilon_t^X \varphi_1 \left(y_t\right)^{1-\varphi_2} \Psi_t\right] \end{aligned}$$

Therefore, first order conditions of this decision problem are given by:

$$y_t : 1 - f\left(\mu_t\right) - \varrho_t - J_t^E \left(1 - \varphi_2\right) \frac{e_t}{y_t} = 0$$

$$\mu_t : -f'\left(\mu_t\right) y_t + J_t^E \frac{e_t}{\left(1-\mu_t\right)} = 0$$

$$e_t : J_t^E = 0$$

$$k_t : \varrho_t \alpha \frac{y_t}{k_t} - r_t = 0$$

$$N_t : \varrho_t \left(1-\alpha\right) \frac{y_t}{N_t} - w_t = 0$$

Note that in a business as usual situation, there is no tax policy:

$$\tau_t = 0.$$

Recall that firms do not consider GHG as a state variable, which implies that in equilibrium cost of carbon J_t^X, as considered by firms, is 0 because firms don't internalize the side effects on emissions on households. As a result, the first order conditions on emissions reads as: $J_t^E = 0$, which in turn implies $\mu_t = 0$.

10.3 Competitive equilibrium under optimal policy

Firms face the following optimization problem:

$$\begin{aligned}
&\max_{\{y_t, N_t, k_t, \mu_t, e_t\}} y_t - w_t N_t - r_t k_t - f\left(\mu_t\right) y_t - \tau_t e_t \\
&+ \varrho_t \left[A_t k_t^\alpha \left(\Gamma_t N_t\right)^{1-\alpha} - y_t\right] \\
&+ J_t^E \left[e_t - \left(1-\mu_t\right) \varepsilon_t^X \varphi_1 \left(y_t\right)^{1-\varphi_2} \Psi_t\right]
\end{aligned}$$

Therefore, first-order conditions of this decision problem are given by:

$$y_t : 1 - f\left(\mu_t\right) - \varrho_t - J_t^E \left(1-\varphi_2\right) \frac{e_t}{y_t} = 0$$

$$\mu_t : -f'\left(\mu_t\right) y_t + J_t^E \frac{e_t}{\left(1-\mu_t\right)} = 0 \tag{29}$$

$$e_t : J_t^E = \tau_t \tag{30}$$

$$k_t : \varrho_t \alpha \frac{y_t}{k_t} - r_t = 0$$

$$N_t : \varrho_t \left(1-\alpha\right) \frac{y_t}{N_t} - w_t = 0$$

Notice first that Equation 26 and Equation 29 imply that $J_t^E = V_t^E$. Then, setting $\tau_t = V_t^X$ in Equation 30 restores the first-best allocation implied by Equation 27. Once the optimal tax is introduced, the first-order conditions obtained under the competitive equilibrium coincide with that of the social planner.

Acknowledgements

This draft has benefitted from comments and suggestions from S. Dietz, an anonymous referee (ECB Working Paper Series), and seminar participants at the ECB.

Ghassane Benmir
London School of Economics and Political Science, London, United Kingdom; email: g.benmir@lse.ac.uk

Ivan Jaccard
European Central Bank, Frankfurt am Main, Germany; email: ivan.jaccard@ecb.europa.eu

Gauthier Vermandel
Paris Dauphine University, Paris, France; email: gauthier.vermandel@dauphine.psl.eu

Postal address 60640 Frankfurt am Main, Germany
Telephone +49 69 1344 0
Website www.ecb.europa.eu

PDF ISBN 978-92-899-4394-9 ISSN 1725-2806 doi:10.2866/48941 QB-AR-20-129-EN-N